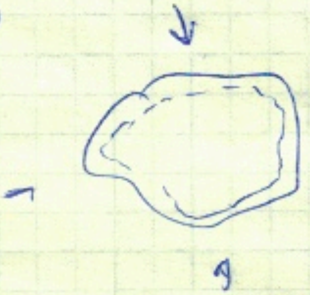
MECHANICS. (Continuum mechanics)

Overview: • Isotropic solids.

Two modes of deformation → volume change (pressure) ①
→ shear deformation ②

①

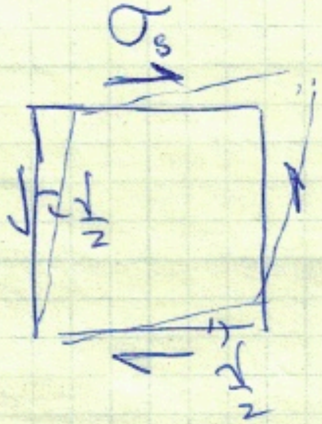


$$p = \left(-\frac{\Delta V}{V}\right) \cdot B$$

B: Bulk m

DOES NOT DEPEND ON SHAPE

②

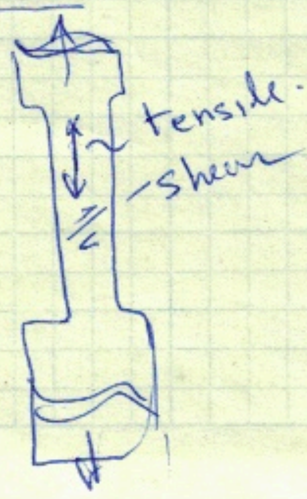


$$\sigma_s = \gamma\, G$$

G: shear modulus

SHAPE CHANGE WITHOUT VOLUME CHANGE.

Experiments

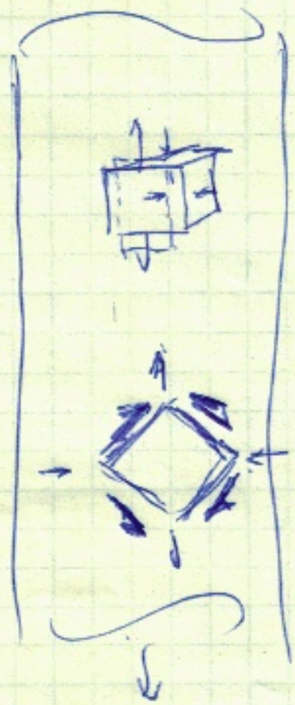


Includes both tensile & shear deformation.

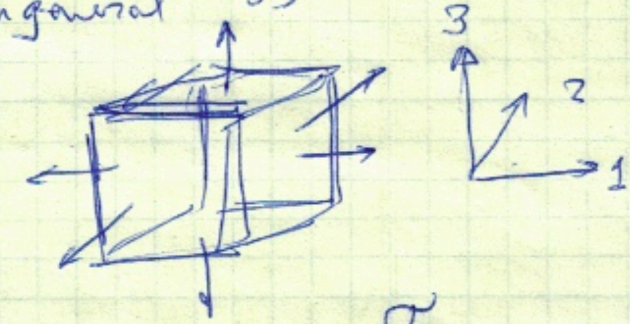
Deformation is a continuum of volumetric & shear Deformation

Description of stress and strain

Uniaxial load

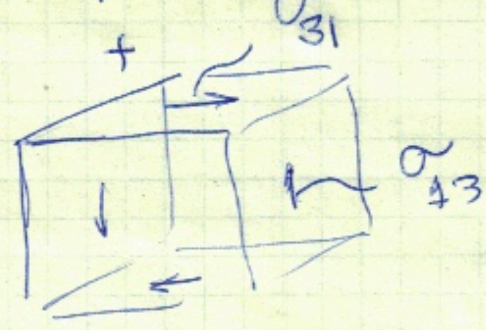


In general σ_{33}

① 

+

σ_{31}

② 

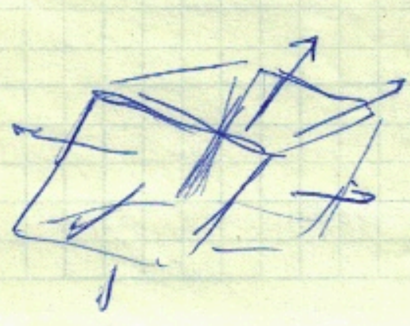
σ_{13}

Plane & direction → tensor.

$$[\sigma] = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$ equal.

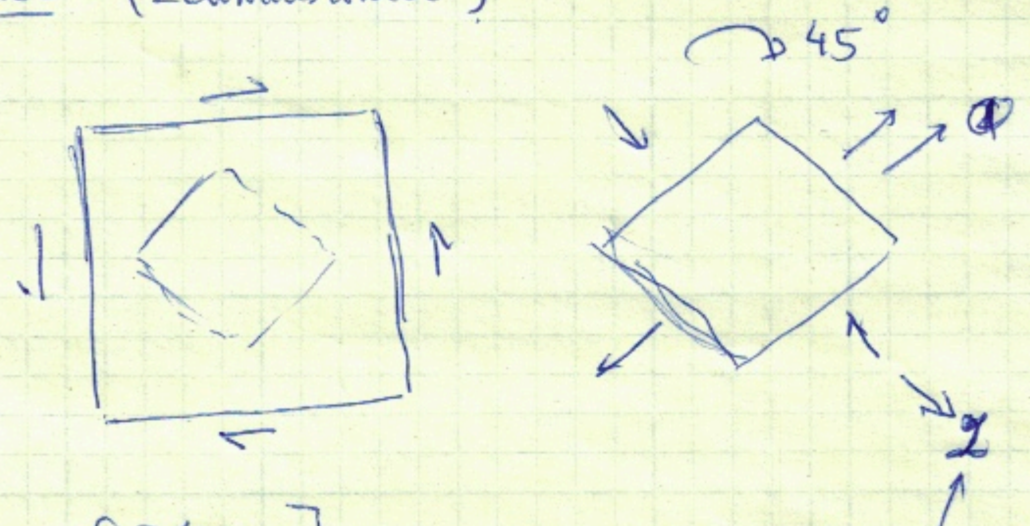
σ_{11}
σ_{22}
σ_{33} six independent components
σ_{12}
σ_{23}
σ_{31}

① + ② } → reorient the cube so that only the forces that are normal to the planes remain → principal axes & principal stresses & principal strains.



$$[\sigma_{ij}] \rightarrow \begin{bmatrix} \sigma_{11}=\sigma_1 & & \\ & \sigma_{22}=\sigma_2 & \\ & & \sigma_{33}=\sigma_3 \end{bmatrix}$$

Pure shear (2 dimensional)

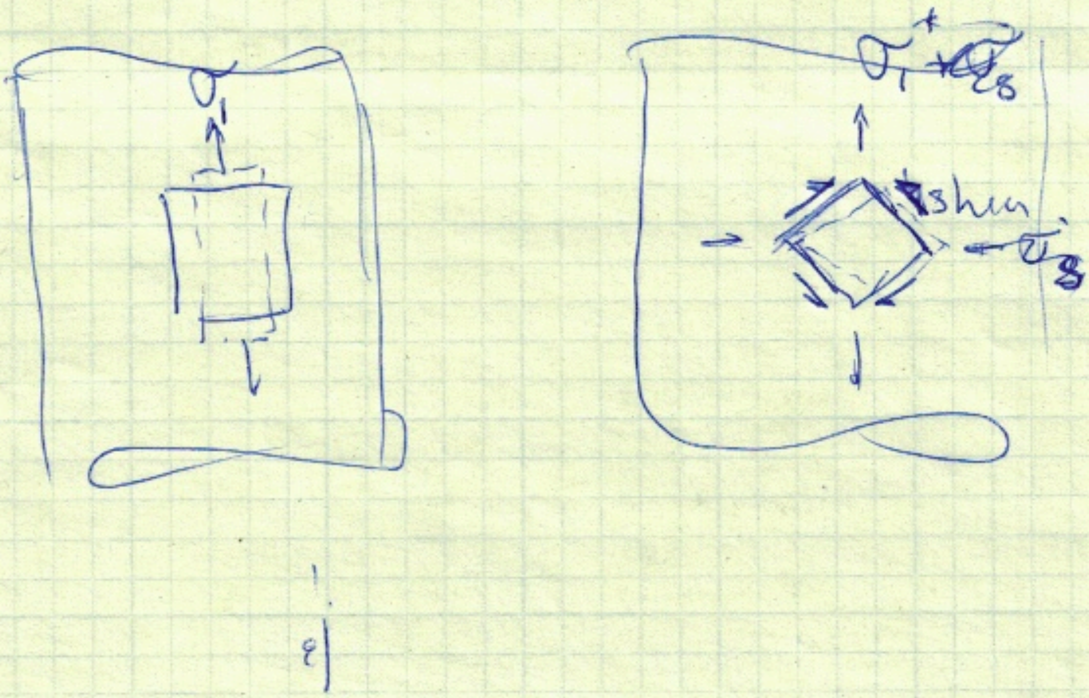


principal axes.

$$[\sigma] = \begin{bmatrix} \sigma = \sigma_1 & & \\ & -\sigma = \sigma_2 & \\ & & 0 \end{bmatrix} \qquad \sigma_s = \frac{\sigma - (-\sigma)}{2} = \sigma$$

Note that $\sigma_1 + \sigma_2 = 0$ (no volume change → pressure = 0)

Uniaxial



$$\begin{bmatrix} \sigma & & \\ & 0 & \\ & & 0 \end{bmatrix} \qquad \sigma_s = \frac{\sigma - 0}{2} = \frac{\sigma}{2}.$$

$$\sigma + 0 + 0 \neq 0$$

(volumetric component)

In general

$$\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} \qquad p = -\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

(hydrostatic press)

$$\text{Shear} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\frac{\sigma_2 - \sigma_3}{2}$$

$$\frac{\sigma_1 - \sigma_3}{2}.$$

Elastic constants

E $\quad \sigma = \epsilon E$

$$\begin{bmatrix} \sigma & & \\ & 0 & \\ & & 0 \end{bmatrix} \qquad \begin{bmatrix} \epsilon_1 & & \\ & \epsilon_2 & \\ & & \epsilon_2 \end{bmatrix}$$

non zero (negative)

$$\nu = -\frac{\epsilon_2}{\epsilon_1}$$

(B) $$-p = B\,(\cancel{\epsilon_1+\epsilon_2+\epsilon_3})\,\frac{\Delta V}{V}$$

$$\frac{\Delta V}{V} = \epsilon_1+\epsilon_2+\epsilon_3$$

$$p = \frac{\sigma_1+\sigma_2+\sigma_3}{3}$$

Pure hydrostatic

$$\sigma_1 = \sigma_1 = \sigma_3 = -p$$

$$\epsilon_1 = \epsilon_2 = \epsilon_3 = \frac{1}{3}\frac{\Delta V}{V}$$

Re Consider E

$$\begin{bmatrix} \sigma & & \\ & 0 & \\ & & 0 \end{bmatrix} \qquad \begin{bmatrix} \epsilon & & \\ & -\nu\epsilon & \\ & & -\nu\epsilon \end{bmatrix}$$

$$p_H = -\frac{\sigma}{3} \qquad \frac{\Delta V}{V} = (1-2\nu)\,\epsilon$$

$$p = B\,\frac{\Delta V}{V}$$

$$-\frac{\sigma}{3} = B\,(1-2\nu)\,E$$

$$B = \frac{E}{3(1-2\nu)}$$

only two elastic constants

E & G

$$\begin{bmatrix} \sigma & & \\ & -\sigma & \\ & & 0 \end{bmatrix} = \begin{bmatrix} \sigma & & \\ & 0 & \\ & & 0 \end{bmatrix} + \begin{bmatrix} 0 & & \\ & -\sigma & \\ & & 0 \end{bmatrix}$$

$\sigma_s = \sigma$ $\quad \gamma =$ $\downarrow$

$$\begin{bmatrix} \epsilon & & \\ & -\nu\epsilon & \\ & & -\nu\epsilon \end{bmatrix} \begin{bmatrix} \nu\epsilon & & \\ & -\epsilon & \\ & & +\nu\epsilon \end{bmatrix}$$

$\sigma = \sigma_s$

$$\begin{bmatrix} (1+\nu)\epsilon & & \\ & -(1+\nu)\epsilon & \\ & & (1+\nu) \end{bmatrix} \text{ zero}$$

$\epsilon_s = \epsilon(1+\nu)$

$\gamma_s = 2\epsilon_s = 2\epsilon(1+\nu)$

$\sigma_s = G\gamma_s$

$\sigma = G \times 2(1+\nu)\epsilon$

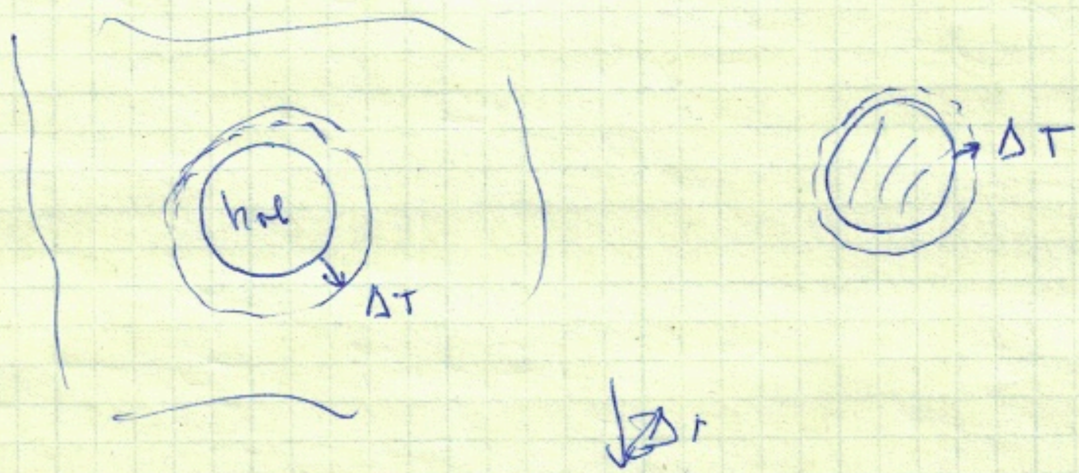
$$G = \frac{E}{2(1+\nu)}.$$

Boundary Value Problems.

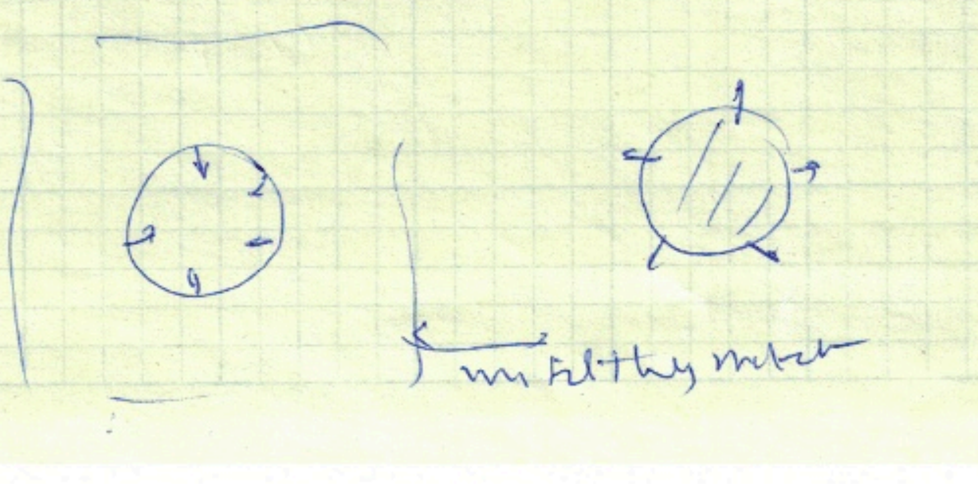
How to use published results ~~...~~ -

~~Exp~~
Example : inclusion within a matrix.

Ceramic within
metal matrix



THE
METHOD



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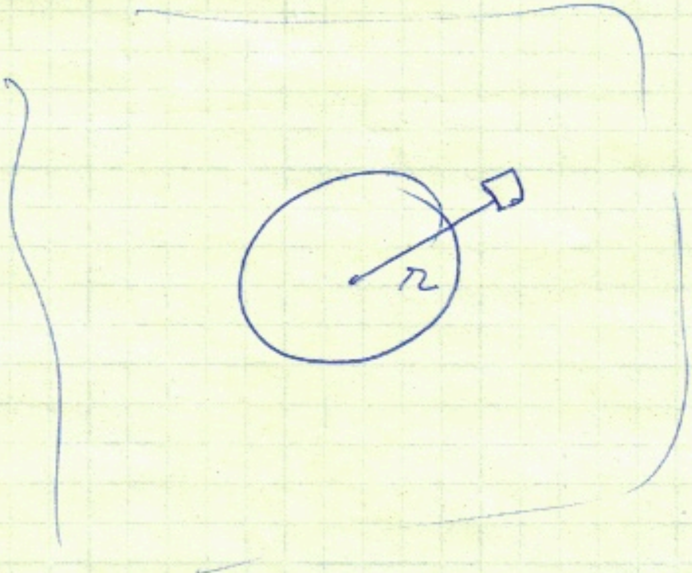
IMDMS

(3)



A, B are consten

$$U_r = \cancel{Ar} + \frac{B}{r^2}$$

$$\sigma_{rr} = -\frac{2E}{(1+\nu)\,r^3} B + \cancel{\frac{E}{(1-2\nu)} A}$$

$$\sigma_{\theta\theta} = \frac{E}{(1+\nu)\,r^3} B + \cancel{\frac{E}{(1-2\nu)} A}$$

$r \to \infty \quad U_r = 0 \quad \sigma_{rr} + \sigma_{\theta\theta} = 0$

$\therefore \; A = 0$

$\sigma_{rr} = -\sigma_{\theta\theta}$ pureshear.

Assume inclusion is rigid

R = inclu rad

$U_r(r=R) = R\,\Delta T \alpha$

$R \cdot \Delta T \alpha = B$

$\sigma_{rr} = \sigma_{\theta\theta} = \frac{E \times R \Delta T \alpha}{(1+\nu) R^3}$

$\sigma_s \propto \frac{1}{R^2} \cdot \frac{E}{(1+\nu)} \cdot \Delta T \cdot \alpha. \quad \frac{\sigma_s max}{r = R}$